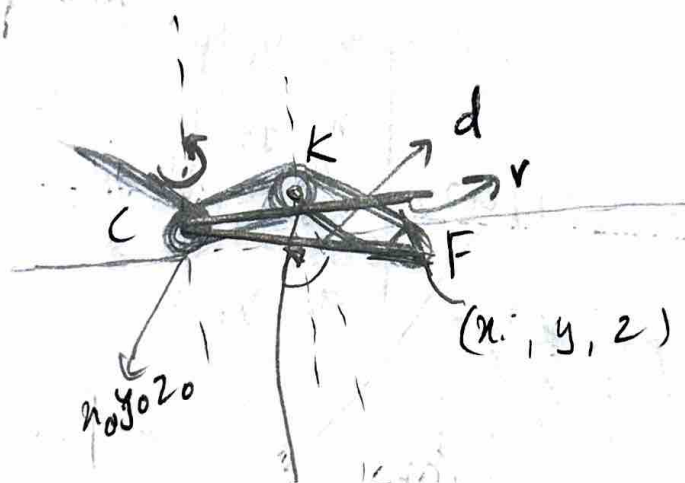
# 3D simulator




foot height relative to ona joint

$z_f$

$x = x_1 + x_0$

$y = y_1 + y_0$

$z = z_1 + z_0$

r

C to F

C = cona (0,0,0) joint

$\theta_c$ = cona angle rotating around vertical axis.

K = knee joint

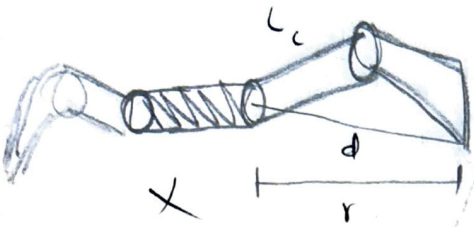
$L_1$ = C to K

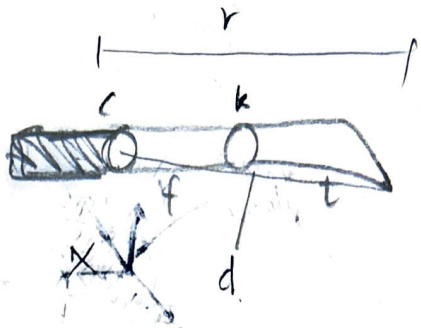
$\theta_k$ = angle of knee servo

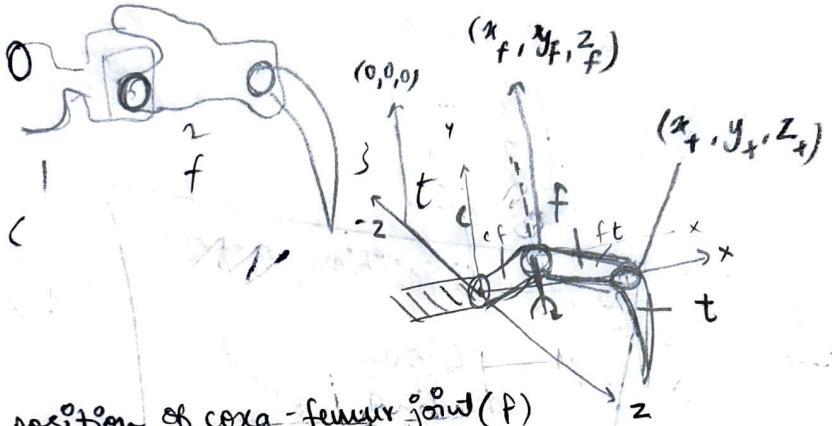
F = end point

$L_2$ = K to F

F = (x, y, z)

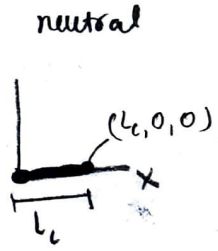


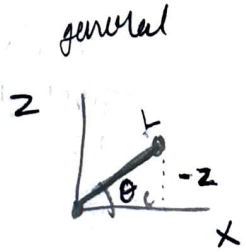

initial position of coxa-femur joint (F)

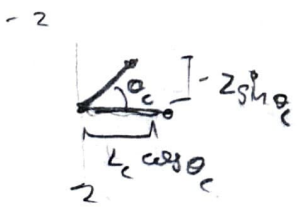
$$P_0 = \begin{bmatrix} L_c \\ 0 \\ 0 \end{bmatrix}$$




general position of coxa - femur joint

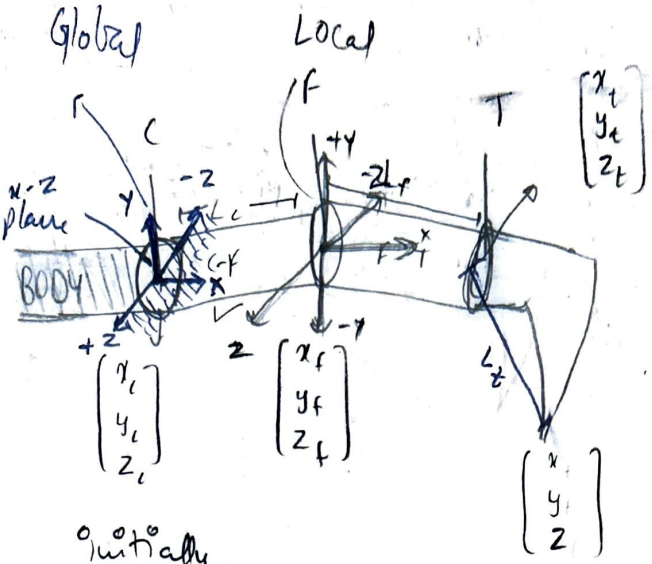
$$P_g = \begin{bmatrix} L_c \cos\theta_c \\ 0 \\ -L_c \sin\theta_c \end{bmatrix}$$




femur - tibia joint

next page



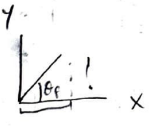

initially,

at neutral, $\theta_f = 0$

$$P_0 = \begin{bmatrix} L_f \\ 0 \\ 0 \end{bmatrix}$$

at general position,

$$P_1 = \begin{bmatrix} L_f \cos\theta_f \\ L_f \sin\theta_f \\ 0 \end{bmatrix}$$

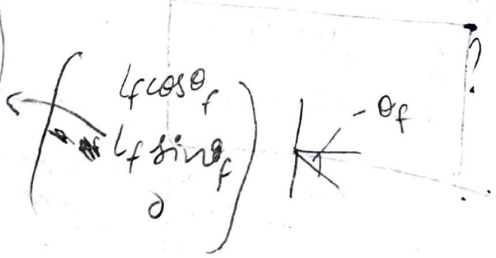



$\eta = \sin\theta_f$

$\frac{\eta}{L_f}$

$\eta = L_f \sin\theta_f$

$\theta_f < 0$




f - t

| angle | axis |
|---|---|
| + | +y |
| − | −y |
| | −30 |

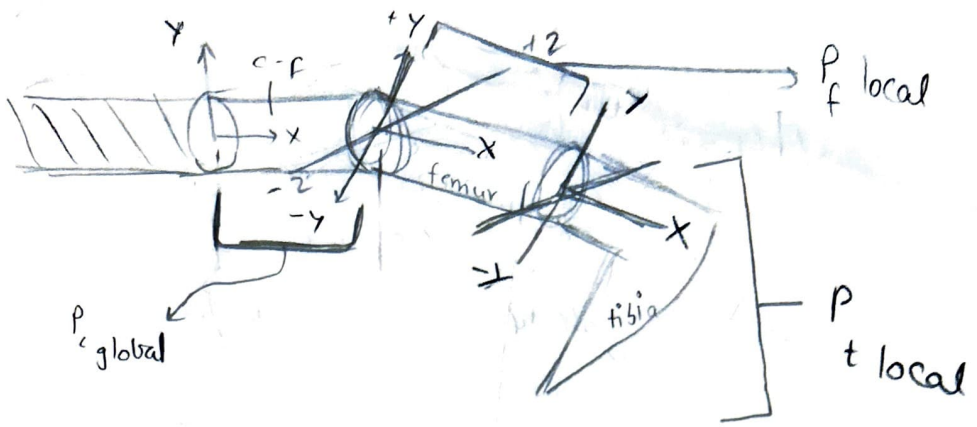
$\sin(-) = -$

$\sin(+) = +$

fixed

fixed

| | angle | axis |
|---|---|---|
| | + | − |
| $-\sin(+) = -$ | − | + |
| $-\sin(-) = +$ | | |




P f local — end of femur link / tibia servo

Hip

$$P_{c\,global} = \begin{bmatrix} L_c \cos\theta_c \\ 0 \\ -L_c \sin\theta_c \end{bmatrix}$$

$$P_{t\,local} = \begin{bmatrix} L_t \cos\theta_t \\ L_t \sin\theta_t \\ 0 \end{bmatrix}$$

foot

$$P_{f\,local} = \begin{bmatrix} L_f \cos\theta_f \\ L_f \sin\theta_f \\ 0 \end{bmatrix}$$

knee

---

transformation of femur's local to global coordinate sys.

$$P_{f\ local} = \begin{bmatrix} L_f \cos\theta_f \\ L_f \sin\theta_f \\ 0 \end{bmatrix}$$ → column matrix [single column & m rows]

tool to coxa-femur join.

$$R_y(\theta_c) = \begin{bmatrix} \cos\theta_c & 0 & \sin\theta_c \\ 0 & 1 & 0 \\ -\sin\theta_c & 0 & \cos\theta_c \end{bmatrix}$$ → Standard right handed rotation matrix around Y.

$$P_{f\,rotated} = R_y(\theta_c) P_{f\,local}$$ → this same as $R_y(\theta_c) R_z(\theta_f) \begin{bmatrix} L_f \\ 0 \end{bmatrix}$

To make it global

$$P_{c}^{global} + P_{f}^{rotated} = P_{f}^{global}$$

$$= \begin{bmatrix} L_f \cos\theta_c \cos\theta_f \\ L_f \sin\theta_f \\ -L_f \sin\theta_c \cos\theta_f \end{bmatrix}$$

femur vector rotated into global orientation

$$\begin{bmatrix} L_f \cos\theta_c \cos\theta_f + L_c \cos\theta_c \\ L_f \sin\theta_f + 0 \\ -L_c \sin\theta_c - L_f \sin\theta_c \cos\theta_f \end{bmatrix} = P_f^{global}$$

$$p_t^{local} = \begin{bmatrix} L_t \cos\theta_t \\ L_t \sin\theta_t \\ 0 \end{bmatrix} \begin{pmatrix} \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix} \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \\ \begin{pmatrix} 1a + 2b + 3c \\ d + 2e + 3f \\ \vdots \end{pmatrix} \end{pmatrix}$$

$$p_{foot}^{global} = p_{knee}^{global} + R_y(\theta_c) R_z(\theta_f) R_z(\theta_t) \begin{bmatrix} L_t \\ 0 \\ 0 \end{bmatrix}_{3\times 1}$$

- $p_{foot}^{global}$ ↙ aka $p_t^{global}$
- $p_{knee}^{global}$ ↓ aka $p_f^{global}$
- $R_y(\theta_c)$ ↙ femur rotation.
- $R_z(\theta_f)$ ↓ $R_z(\theta_f + \theta_t)$
- $R_z(\theta_t)$ ↘ tibia rotation
- $3\times 1$

$$p_{foot}^{global} = \begin{bmatrix} \cos\theta_c \left[ L_c + L_f \cos\theta_f + L_t \cos(\theta_f + \theta_t) \right] \\ L_f \sin\theta_f + L_t \sin(\theta_f + \theta_t) \\ -\sin\theta_c \left[ L_c + L_f \cos\theta_f + L_t \cos(\theta_f + \theta_t) \right] \end{bmatrix}$$

let's verify